Math 5 SN Midyear Review

1 A microbiologist is studying two bacteria populations.

Last Monday, the 1st population numbered 2000 and the 2nd numbered 2 048 000.

He noted that the 1st population doubled every day while the 2nd population was reduced by half each day.

After how many days would the two populations of bacteria be equal in number?

2 At last count, a certain species of bird numbered 200, raising fears that its extinction was imminent. Hunting this species is now forbidden. With this law in place, biologists claim that the population of this species will double every 6 months.

They believe that the species will be saved once its population reaches 18 500.

In how many years will this species no longer be threatened with extinction?

3 Three years ago Greg invested $1000 at a fixed interest rate compounded every 6 months. His investment is currently valued at $1400.

Given $C_n = C_0 \left(1 + \dfrac{t}{k}\right)^{nk}$ where

C_n is the capital after n years
C_0 is the capital invested
t is the annual interest rate
k is the number of times per year that interest is paid
n is the number of years

To the nearest hundredth of a percent, what is the annual rate of interest?

4 The value v, in dollars, of a stock varies with time t, in days. This situation is defined by

$$v(t) = -\left|\dfrac{t}{2}\right| + 5 \quad \text{if} \quad t \in [0, 6],$$

$$v(t) = \dfrac{5}{2}\sqrt{t-6} + 2 \quad \text{if} \quad t \in [6, 10],$$

$$v(t) = -0.6t + 13 \quad \text{if} \quad t \in [10, 20].$$

For how many days was the value v of the stock $3.75 or less?

5 For a science project, Louise and Bob recorded the temperature outside their school for a 24 hour period. At the beginning, the thermometer read -1 °C. The minimum temperature of -5 °C was reached 8 hours later.

The data recorded show that the temperature t(x) varied as a function of the number of hours x elapsed and that t is an absolute value function.
During how many hours was the temperature less than or equal to -3 °C?

6 Computers have changed a great deal since 1950 because of the miniaturization of the circuits.

Year	Number of circuits on a chip $f(x)$
1950	35
1960	3 500
1970	350 000
1980	35 000 000
1990	....

The function representing this situation is given by
$$f(x) = 35(10^{wx})$$

where x represents the number of years since 1950 and w, a parameter.
What is the value of the parameter?

7 A basket of groceries today costs $200. If the rate of inflation remains at 4 % for the next few years, how much will the same grocery basket cost in 5 years?

Note : Express your answer to the nearest hundredth.

8 A radioactive substance disintegrates at a rate such that after 2 years it has $\dfrac{4}{9}$ of its initial mass. If you have 60 grams of this substance, how much of it will remain after 12 years?
Note : Express your answer in grams to the nearest hundredth.

9 During a fireworks show, two rockets must be launched from two different spots at the same time.

The heights, in metres, of the two rockets as a function of time t, in seconds, are represented by the following equations:
$$h_1(t) = -12.5(t - 4)^2 + 200$$
$$h_2(t) = 25\sqrt{\frac{t}{d}} + 50$$

Both rockets must explode 6 seconds after they have been launched. They must also explode at the same height.
What is the value of parameter d?

10 One very hot summer day, the outside temperature is described by the following rule:
$$T(x) = -3\,|\,x - 6\,| + 36$$
where x represents the number of hours that have elapsed since sunrise
       $T(x)$ represents the temperature in ^{o}C

To make the temperature inside a shopping centre more comfortable, an air conditioning system starts up when the outside temperature reaches 21 °C and stops when the outside temperature drops to 20 °C.

How long was the air conditioning system in operation on that day?

11 The value of a certain amount of capital C_o invested at a given rate of interest i for t years will be as follows:

$$C(t) = C_o(1 + i)^t$$

Emily invested the $2000 she received for winning a design contest. This money will earn the same rate of interest throughout the term of the investment.

The following table of values shows the value of Emily's investment as a function of time in years.

Term of the Investment (years)	Value of the Investment ($)
1	2200
3	2662

How much will Emily's investment be worth after 10 years?

12 Jenny and Eric analyzed the changes in the value of Future Telecom's shares in 1999. On January 1st 1999, the initial value of a share was $25. On may 31st, the share reached its minimum value of $10. Since then, the value of shares has been on the increase.
They noticed that the relation between the elapsed time, in months, and the value of a share, in dollars, was an absolute value function.

What was the value of a share on December 31st, 1999?

13 The number of people living in Kilwat, Germany, varies according to the rule of an exponential function. On January 1st 1975, the city's population was 130 000. On January 1st 1985, it was 260 000. What was the population of this German city on January 1st 2000, given that the growth rate remained constant?

14 A family of four spent $200.00 a week on groceries in 2001.

When will the weekly grocery bill for a family of four be $250.00 if the cost of living increases by 3% per year?
Round your answer to the nearest tenth.

15 Sonia is replacing some of the water in her aquarium without removing the fish.

When she started working, the aquarium contained 60 litres. After 35 seconds, 45 litres of water remained in the aquarium. This is the minimum quantity of water needed to ensure that the fish will not be harmed. Then, using a pump, Sonia filled the aquarium to its maximum capacity of 70 litres.

Sonia noted that the relation between the time elapsed, in seconds, and the quantity of water in the aquarium, in litres, corresponds to an absolute value function.

Throughout this process, for how many seconds was the quantity of water in the aquarium less than 53 litres?

16 Two missiles are launched 2 seconds apart. The paths they follow over a span of 8 seconds can be represented by two different square root functions, as illustrated below:
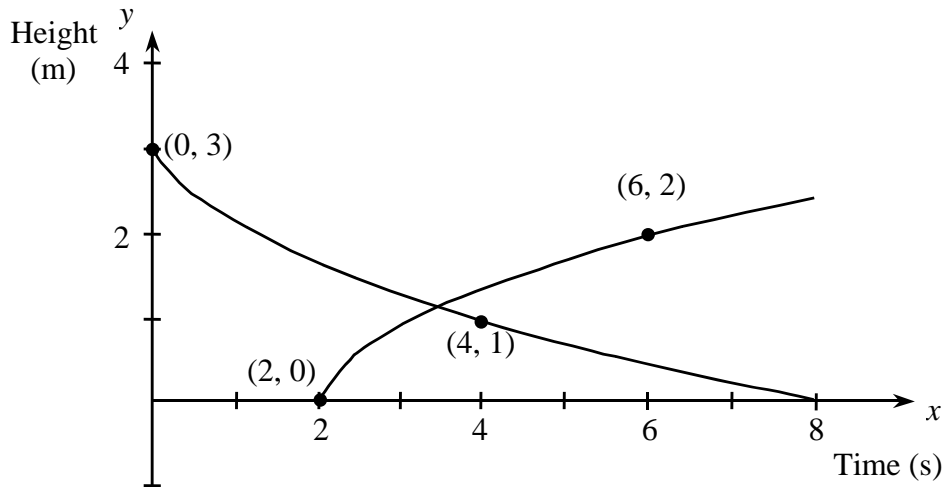

How many seconds after the 2nd projectile has been launched, will it be higher than the 1st projectile?

17 A car's speed is a function of time, which can be represented by an absolute value function graph. Dean's car goes from rest to a maximum speed of 150 km/h in 12 seconds and slows down at the same rate when he brakes.

For how many seconds is the speed of Dean's car at least 50 km/h but no more than 120 km/h?

18 The graph shown below, with the axes graduated in meters, represents a slide that spans 31 metres in length.
Three posts placed 8 meters apart support the first part of the slide. This part of the slide is represented by a square root function with vertex $(0, 15)$.

The second part of the slide is linear and forms an angle of 25° with the horizontal.
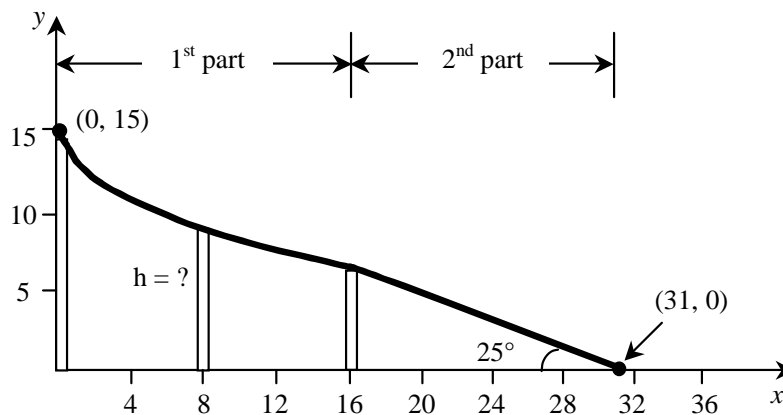
As shown in the graph above, the second post is 8 meters away from the first, which is located on the y-axis.
What is the height of the second post? Round your answer to the nearest hundredth of a metre.

19 Jonathan works at a golf club during his summer vacation. He sometimes cleans the premises and sometimes works in the kitchen at the club's restaurant.
Jonathan makes $8 per hour when cleaning the premises and $9.50 per hour when working in the kitchen.

There are certain constraints on the number of hours he can devote to each job every week. This situation is represented by the system of inequalities and the polygon of constraints given below.

$$x \geq 0$$
$$y \geq 0$$
$$x + y \leq 40$$
$$x \geq 16$$
$$y \leq 20$$

x : the number of hours spent cleaning the premises
y : the number of hours spent working in the kitchen
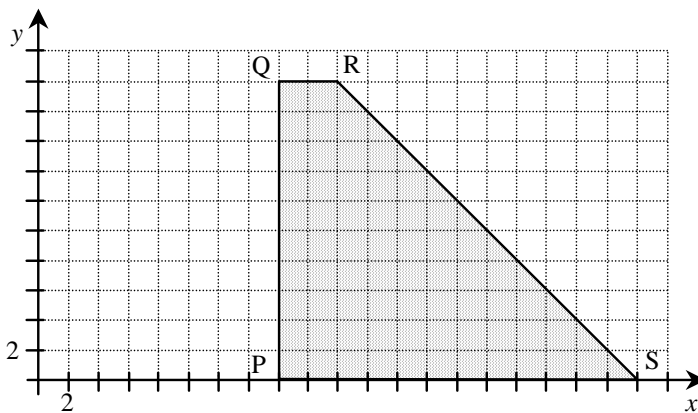
Coordinates of the vertices of the polygon
P(16, 0)
Q(16, 20)
R(20, 20)
S(40, 0)

This week, Jonathan's employer informed him that there would be an additional constraint. This new constraint is represented by the following inequality:

$$x \geq y + 20$$

With this new constraint, by how much will Jonathan's maximum possible income decrease?

20 When interest is paid n times a year, the value of a certain amount of capital C_0 invested at an annual interest rate i for t years will be as follows:

$$C(t) = C_0 \left(1 + \frac{i}{n} \right)^{nt}$$

Gerry wants to invest $2 000 for 2 years. He has two investment options.

| Investment option A	Investment option B
Annual interest rate of 5% Interest paid once a year	Annual interest rate of 4.2% Interest paid 12 times a year

Gerry chose investment option A because he was told it would provide the best return.

Rounded to the nearest whole month, how many months would Gerry have had to invest his money under investment option B in order to earn the same amount he will earn under investment option A?

21 To raise money, the Graduation Committee decides to sell cases of fruit. The following polygon represents the constraints that must be respected.

If x represents the number of cases of oranges for sale and y, the number of cases of grapefruit for sale, the constraints are:
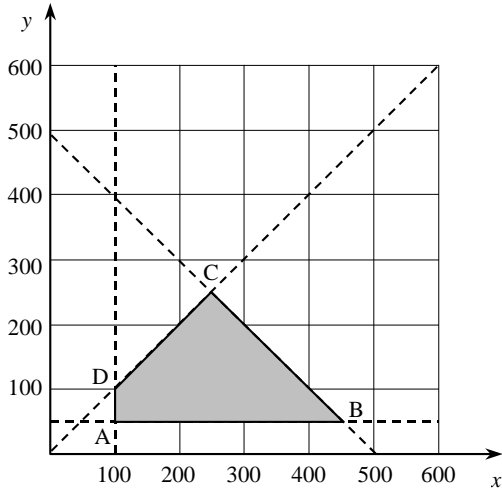
① $x \le y$
② $x + y \le 500$
③ $x \ge 100$
④ $y \ge 50$

For each case of oranges and grapefruit sold, the Graduation Committee makes a profit of $1.00 and $1.50, respectively.

Yesterday, the head of the committee received a call from the supplier. Because of a recent flood, the supplier can deliver a maximum of 400 cases of fruit.
By how much will the maximum possible revenue decrease because of the flood?

22 Wheeler is a producer of mountain bikes and road bikes. Because of its small size, it can build no more than 80 bikes each week. To meet certain conditions in its workshop, it must build at least 45 mountain bikes, and at least 10 road bikes weekly. To meet consumer demand, it must manufacture at least 3 times as many mountain bikes as road bikes.

The following is the system of constraints for Wheeler's weekly bike production:

$x =$ the number of road bikes produced weekly
$y =$ the number of mountain bikes produced weekly

$$x \ge 0$$
$$y \ge 0$$
$$x \ge 10$$
$$y \ge 45$$
$$x + y \le 80$$
$$y \ge 3x$$

For each road bike and mountain bike produced, Wheeler earns a profit of $250 and $175, respectively.
What is the maximum weekly profit that can be earned?

23 On her 21st birthday, Marie received a lump sum of money. At that time, she decided to invest it at a fixed annual interest rate, compounded yearly, until her 30th birthday.

On her 25th birthday, her investment had grown exponentially to $11 360.08. On her 30th birthday, it had further grown to $16 691.69.

This situation is represented in the graph below.
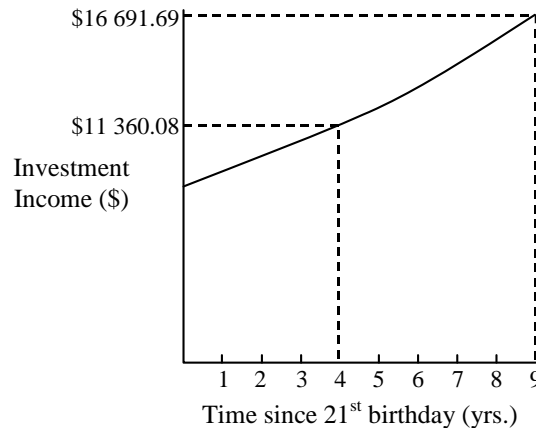
Rounded to the nearest tenth of a percent, what was the fixed annual interest rate over this nine-year period?

24 A fisherman has to separate his daily catch of shellfish into two categories before he can sell them. Lobsters are sold for $8.70 each and crabs are sold for $9.60 each.

On an average day, the fisherman can expect to catch a minimum of 35 crabs and a maximum of 60. By experience, there are at most twice as many lobsters as crabs in a daily catch and never has the fisherman caught more than 140 shellfish in a single day.

Using a polygon of constraints, determine the maximum revenue that this fisherman can expect to make.

25 A tennis ball is hit by a racket from a height of 2 metres and follows the path of an absolute value function. One second later the ball hits the ceiling, which has a height of 10 metres. On its way down, the ball bounces off a table that is 1 metre high. After the bounce, its path is a semi parabola. One second after the ball hits the table, it reaches a height of 3 metres before hitting a wall at a height of 5 metres.

How many seconds after the ball was hit by the racket did it hit the wall?
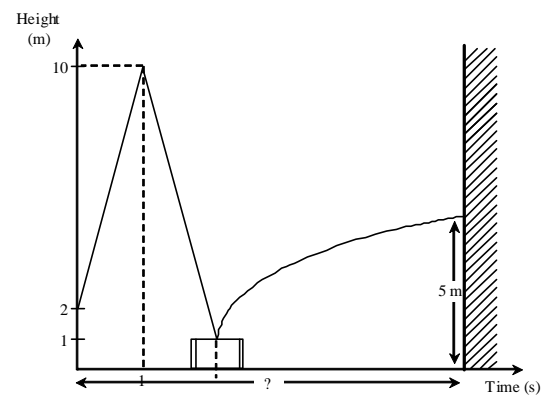
Math 5SN – Midyear Review Answers

| 1 | Result : The two populations will be equal in number after 5 days. |

| 2 | Answer : This species of birds will no longer be threatened with extinction in about 3.3 years. |

| 3 | Answer : The annual rate of interest is 11.52 %. |

| 4 | Result : The value v of the stock was \$3.75 or less for approx 19.41 days (2911/150 days). |

| 5 | Result : The length of time the temperature was less than or equal to -3 °C was 8 hours. |

| 6 | Result : The parameter is 0.2. |

| 7 | Result : \$243.33 |

| 8 | Result : 0.46 grams |

| 9 | Answer   The value of parameter d is $\frac{3}{8}$. |

| 10 | Answer   On that day, the air conditioning system was in operation for 10 hours and 20 minutes. |

| 11 | Answer   Emily's investment will be worth \$5187.48 after 10 years. |

| 12 | Answer   On December 31st 1999, the share's value was \$31. |

| 13 | Answer   On January 1st 2000, there were 735 391 people in Kilwat. |

| 14 | Answer: In 7.5 years, the cost of weekly groceries for a family of four will be \$250. |

| 15 | Answer: The quantity of water in the aquarium is less than 53 litres for 37.3 seconds. |

| 16 | Answer: 1.36 seconds after it has been launched, the 2nd projectile will be higher than the 1st projectile. |

| 17 | Answer: The car's speed will be at least 50 km/h but no more than 120 km/h for 11.2 seconds. |

| 18 | Answer: The height of the second post is 9.34 meters. |

19 Maximum possible income before the new constraint

Vertex	Income: $8x + 9.50y$
P (16, 0)	8 (16) + 9.50 (0) = $128
Q (16, 20)	8 (16) + 9.50 (20) = $318
R (20, 20)	8 (20) + 9.50 (20) = $350 ← maximum income
S (40, 0)	8 (40) + 9.50 (0) = $320

Maximum possible income with the new constraint

Vertex	Income: $8x + 9.50y$
(20, 0)	8 (20) + 9.50 (0) = $160
(30, 10)	8 (30) + 9.50 (10) = $335 ← maximum income
S (40, 0)	8 (40) + 9.50 (0) = $320

Difference between the two maximum possible incomes

$350 − $335 = $15

Answer: With this new constraint, Jonathan's maximum possible income will decrease by $**15**.

20   Answer: Gerry would have had to invest his money for **28** months, to the nearest whole month, under investment option B in order to earn the same amount he will earn under investment option A.

21   Answer: The decrease in revenue caused by the flood is $**125**.

22
Points (x, y)	Calculation	Profit
1. (10, 45)	250(10) + 175(45)	$10 375
2. (15, 45)	250(15) + 175(45)	$11 625
3. (20, 60)	250(20) + 175(60)	$15 500
4. (10, 70)	250(10) + 175(70)	$14 750

Answer   The maximum weekly profit is $15 500.

23   Answer   The annual fixed interest rate is approximately 8%.

24   Answer: The maximum revenue this fisherman can expect to make is $**1272**.

25   Answer: The ball hits the wall **6.125** seconds after it was hit by the racket.